Broadside, Edge-Coupled, Symmetric Strip Transmission Lines

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Abstract —**This paper analyzes the propagation parameters of the general, shielded, broadside, edge-coupled, symmetric strip transmission line using the transverse transmission line method combined with the variational technique in the space domain. A simple approximation to the charge distribution on the conducting strip is assumed. Extensive design data on the characteristic impedances and effective dielectric constants are generated for a) broadside, edge-coupled, homogeneous striplines, b) broadside, edge-coupled, microstrip lines, and c) broadside, edge-coupled, microstrip lines with inverted dielectric. The effect of the shielding side walls on the characteristic impedances is investigated. The results presented should find application in the design and fabrication of directional couplers and filters having complex electric responses.**

I. Introduction

Microwave circuits used in a communication system extensively use coupled-line structures as the basic building blocks for directional couplers, filters, and various other important devices [1]–[3]. Depending on the excitation, a pair of coupled lines can support two propagation modes, namely, the even-mode and the odd-mode. Such coupled lines in various microstrip-like configurations yield small deviations from equal-mode phase velocities [4], [5]. Large phase velocity deviations can be obtained in inhomogeneous broadside-coupled striplines [6], [7]. It has been shown that coupled lines which offer large phase velocity deviations can be used to design some interesting filters [7] and directional couplers [8]. Various two-port coupled-line configurations in a homogeneous as well as an inhomogeneous medium are reported in the literature [9], [10]. The electrical response of these coupled-line structures have been studied using the ABCD and the impedance matrix formulations [9]–[11]. The matrix formulation technique requires a prior knowledge of the even- and odd-mode characteristic impedances and the effective dielectric constants. For a pair of coupled lines in a homogeneous as well as an inhomogeneous medium, extensive data on the impedance characteristics are available in the literature [4]–[7].

In recent years, there has been considerable interest in multiconductor coupled transmission lines in view of their application in a variety of MIC components. Some of the examples are: three-line microstrip couplers [12], interdigital or Lange couplers [13], [14], and meander folded couplers [15]. Depending on the terminal conditions, these coupled transmission lines can support several propagating modes. In order to study the electric response produced by these coupled transmission lines, for various excitations, a knowledge of the characteristic impedances and the effective dielectric constants for all the possible propagating modes is essential. Several authors have reported the impedance characteristics of multiconductor transmission lines using various analytical as well as the network analysis techniques [12]–[18]. Recently, impedance characteristics of homogeneous, shielded, broadside, edge-coupled, strip transmission lines (Fig. 1 with $\epsilon_{r1} = \epsilon_{r2} = 1$) have been reported [18]. Its inhomogeneous counterpart is also reported in the literature [16]. However, no design data on the impedance characteristics and effective dielectric constants are available.

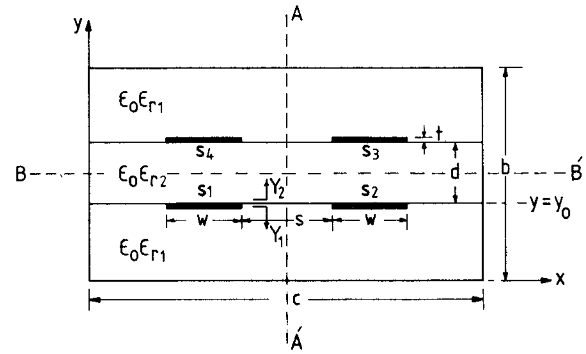

Fig. 1 A schematic of three-layer, broadside, edge-coupled stripline.

In this paper, the three-layer, broadside, edge-coupled, strip transmission line is analyzed using the transverse transmission line method combined with the variational technique in the space domain. The characteristic impedances and the effective dielectric constants of all the possible propagating modes are computed for a) broadside, edge-coupled, homogeneous striplines, b) broadside, edge-coupled, microstrip lines, and c) broadside, edge-coupled, microstrip lines with inverted dielectric. The effect of shielding side walls on the impedance characteristics is investigated. The impedance and effective dielectric constant data presented should be useful in designing directional couplers and filters with complex electric responses.

II. Analysis

The schematic diagram of the three-layer, broadside, edge-coupled, transmission line to be analyzed is shown in Fig. 1. This structure can support four propagation modes

Manuscript received April 1, 1982; revised June 21, 1982.

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[18]

1) even–even mode (ee): AA' magnetic wall, BB' magnetic wall
2) even–odd mode (eo): AA' magnetic wall, BB' electric wall
3) odd–even mode (oe): AA' electric wall, BB' magnetic wall
4) odd–odd mode (oo): AA' electric wall, BB' electric wall.

To compute the characteristic impedances, it is sufficient to analyze only one quarter of the structure with appropriate boundary conditions at $x = c/2$ and $y = b/2$, corresponding to four different modes.

The characteristic impedance (Z) and the effective dielectric constant (ϵ_{eff}) of a general TEM transmission line can be written as

$$Z = \frac{1}{v_0\sqrt{CC_0}} \tag{1}$$

$$\epsilon_{\text{eff}} = \frac{C}{C_0} \tag{2}$$

where C and C_0 are the line capacitances of the transmission line with and without the dielectrics, respectively, and v_0 is the propagation velocity of electromagnetic energy in free space. The line capacitances for each mode can be obtained from the variational expression [19]

$$C = \frac{\left[\int_{s_1} f(x)\,dx\right]^2}{\int_{s_1}\int_{s_1} G(x, y_0/x_0, y_0) f(x) f(x_0)\,dx\,dx_0}. \tag{3}$$

Here, $f(x)$ is the charge distribution on the conducting strip s_1 and $G(x, y_0/x_0, y_0)$ is the Green's function at the charge plane $y = y_0$. This Green's function is the solution of the Poisson's equation

$$\nabla^2 G(x, y/x_0, y_0) = -\frac{1}{\epsilon}\delta(x - x_0)\delta(y - y_0) \tag{4}$$

where $\delta(x - x_0)$ and $\delta(y - y_0)$ are the Dirac's delta functions and ϵ is the absolute dielectric constant of the medium containing the charge. For the present problem, applying the transverse transmission line method [5], [20], the Green's function for all the four modes can be expressed as

$$G(x, y_0/x_0, y_0)_{\begin{pmatrix}\text{ee}\\ \text{eo}\\ \text{oe}\\ \text{oo}\end{pmatrix}} = \sum_{n\begin{pmatrix}\text{odd}\\ \text{odd}\\ \text{even}\\ \text{even}\end{pmatrix}} \frac{4}{n\pi Y_{\begin{pmatrix}\text{ee}\\ \text{eo}\\ \text{oe}\\ \text{oo}\end{pmatrix}}} \sin(\beta_n x_0)\sin(\beta_n x);\ \beta_n = \frac{n\pi}{c} \tag{5}$$

where Y_{ee}, Y_{eo}, Y_{oe}, and Y_{oo} are the admittances at the charge plane $y = y_0$ for the four propagating modes.

The admittance Y at the charge plane $y = y_0$(Fig. 1.) for a specific mode is given by

$$Y = Y_1 + Y_2 \tag{6}$$

where Y_1 and Y_2 are the admittances at $y = (b - d)/2$ looking into the negative and positive y directions, respectively. Using the standard expression to obtain the admittance of a section of transmission line, we get

$$Y = \epsilon_0\left[\epsilon_{r1}\coth\frac{n\pi(b-d)}{2c} + \epsilon_{r2}\left(\tanh\frac{n\pi d}{2c}\right)^P\right]. \tag{7}$$

Here, $P = 1$ for ee and oe modes, and $P = -1$ for eo and oo modes.

In order to evaluate the line capacitances, the charge distribution $f(x)$ has to be specified for each mode. It has been shown in an earlier paper [5] that a charge distribution of the type given below yields fairly accurate values of the capacitance for coupled microstrip-like transmission lines

$$f(x)\begin{cases}\dfrac{1}{w}\left[1 + \left|\dfrac{2}{w}\left(x - \dfrac{c-s-w}{2}\right)\right|^3\right]; \\ \qquad \dfrac{c-s}{2} - w \leqslant x \leqslant \dfrac{c-s}{2} \\ 0;\ \text{otherwise.}\end{cases} \tag{8}$$

Substituting (5) and (8) in (3), and evaluating the integral, we obtain

$$C_{\begin{pmatrix}\text{ee}\\ \text{eo}\\ \text{oe}\\ \text{oo}\end{pmatrix}} = \frac{1.5625}{\sum_{n\begin{pmatrix}\text{odd}\\ \text{odd}\\ \text{even}\\ \text{even}\end{pmatrix}} (L_n + M_n)^2 P_n / Y_{\begin{pmatrix}\text{ee}\\ \text{eo}\\ \text{oe}\\ \text{oo}\end{pmatrix}}} \tag{9a}$$

where

$$P_n = \frac{4}{n\pi}\left(\frac{2}{\beta_n w}\right)^2; \qquad \beta_n = \frac{n\pi}{c} \tag{9b}$$

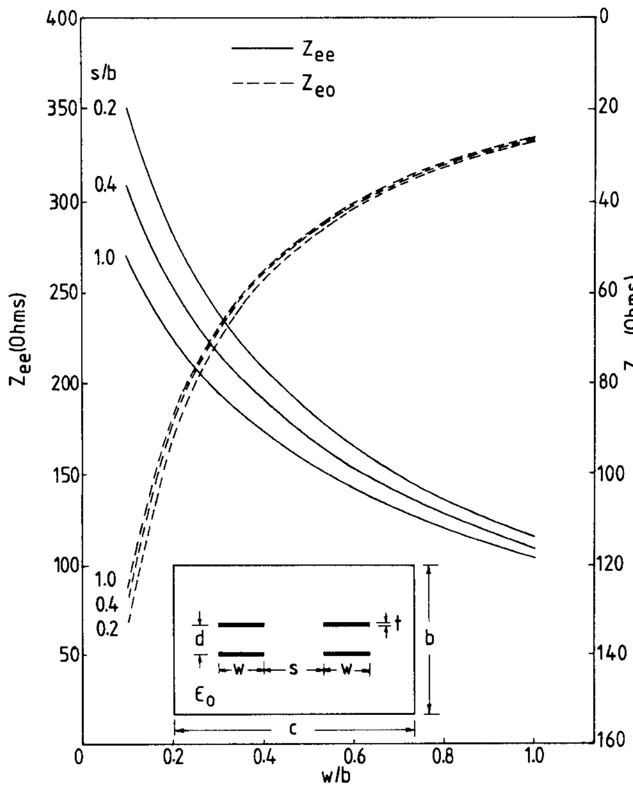
$$L_n = \sin\left(\frac{\beta_n w}{2}\right)\sin\left(\frac{\beta_n(c-s-w)}{2}\right) \tag{9c}$$

$$M_n = \left(\frac{2}{\beta_n w}\right)^3 \sin\left(\frac{\beta_n(c-s-w)}{2}\right)\left[3\left\{\left(\frac{\beta_n w}{2}\right)^2 - 2\right\} \cdot\cos\left(\frac{\beta_n w}{2}\right) + \left(\frac{\beta_n w}{2}\right)\left\{\left(\frac{\beta_n w}{2}\right)^2 - 6\right\}\sin\left(\frac{\beta_n w}{2}\right) + 6\right] \tag{9d}$$

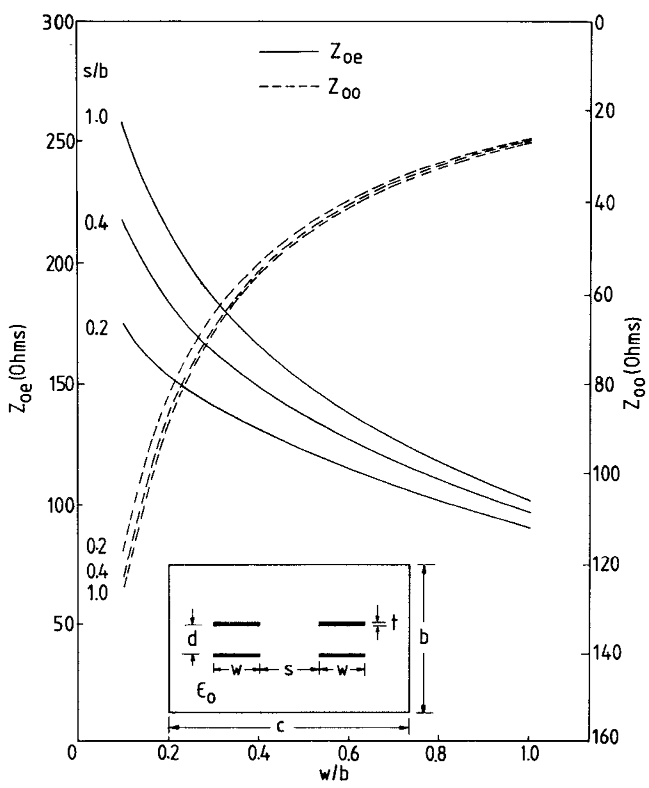
and Y_{ee}, Y_{eo}, Y_{oe}, and Y_{oo} are given by (7).

III. Numerical Results

Numerical results for the line capacitances of the broadside, edge-coupled, striplines are calculated by evaluating (9). The characteristic impedances and the effective dielectric constants are then obtained from (1) and (2). The effects of the shielding side walls on the computed characteristic impedances have been investigated. The value of c/b is varied from 1 to 10. It is observed that the even–even and the odd–even mode impedances (Z_{ee}, Z_{oe}) increase as the aspect ratio c/b is increased. The change in the even–odd and the odd–odd mode impedances (Z_{eo}, Z_{oo})

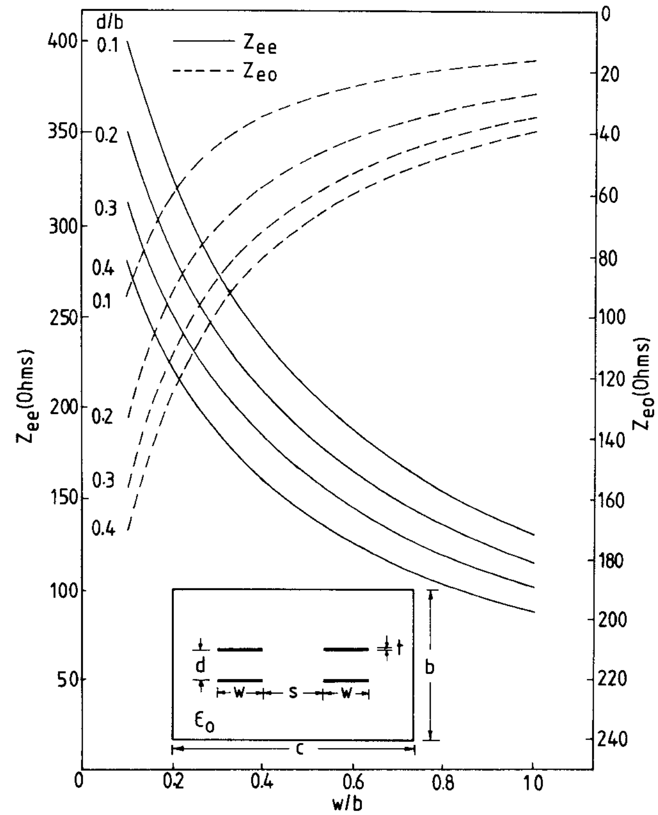


(a)

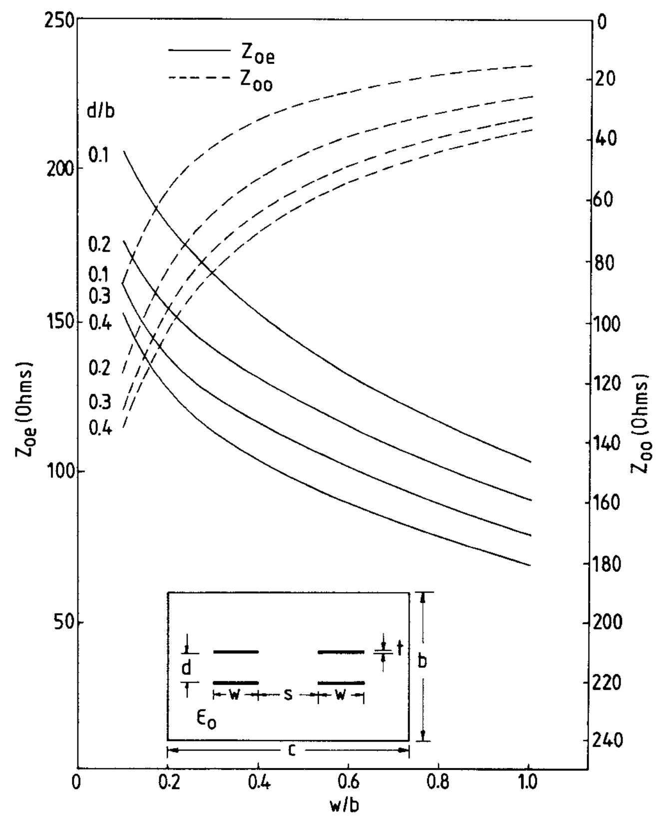


(b)

Fig. 2. The computed characteristic impedances of the broadside, edge-coupled, stripline with air dielectric, for a fixed broadside spacing $d, t/b = 0, c/b = 10, d/b = 0.2$. (a) Even–even and even–odd mode impedances. (b) Odd–even and odd–odd mode impedances.



(a)



(b)

Fig. 3. The computed characteristic impedances of the broadside, edge-coupled, stripline with air dielectric, for a fixed edge spacing $s, t/b = 0, c/b = 10, s/b = 0.2$. (a) Even–even and even–odd mode impedances. (b) Odd–even and odd–odd mode impedances.

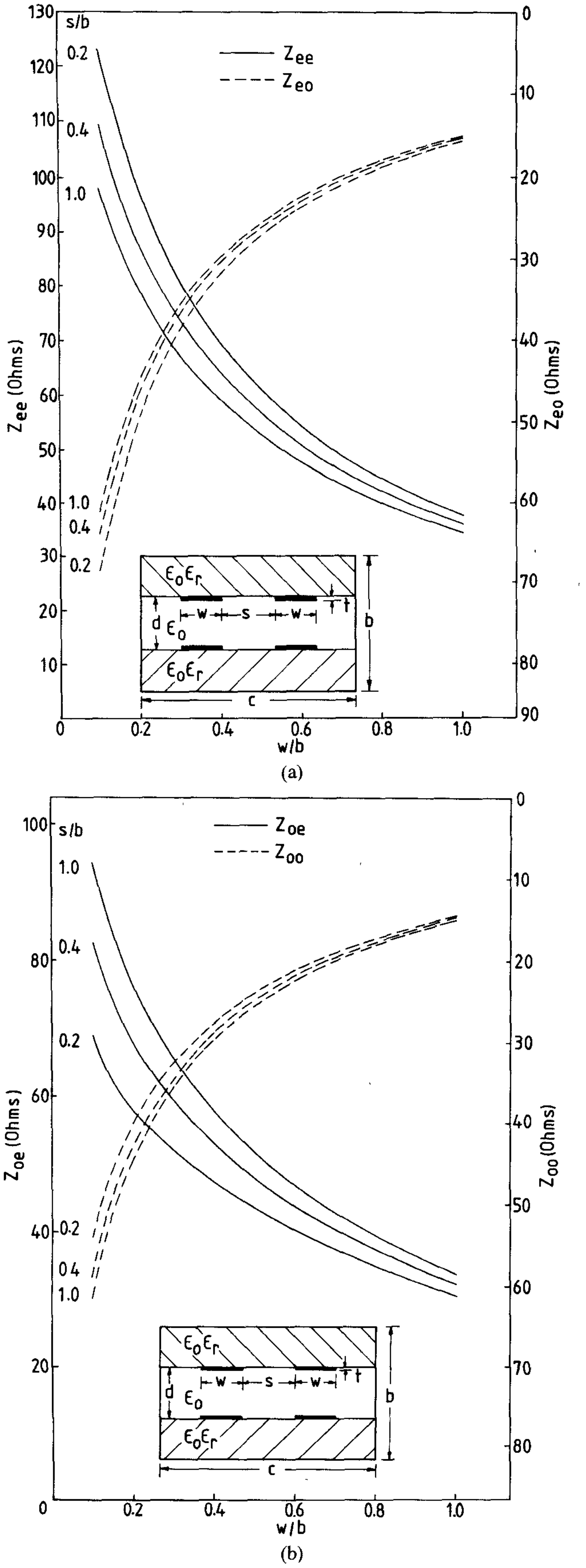


Fig. 4. The computed characteristic impedances of the broadside, edge-coupled, microstrip, for a fixed broadside spacing $d, t/b = 0, c/b = 10, d/b = 0.2, \epsilon_r = 10$. (a) Even–even and even–odd mode impedances. (b) Odd–even and odd–odd mode impedances.

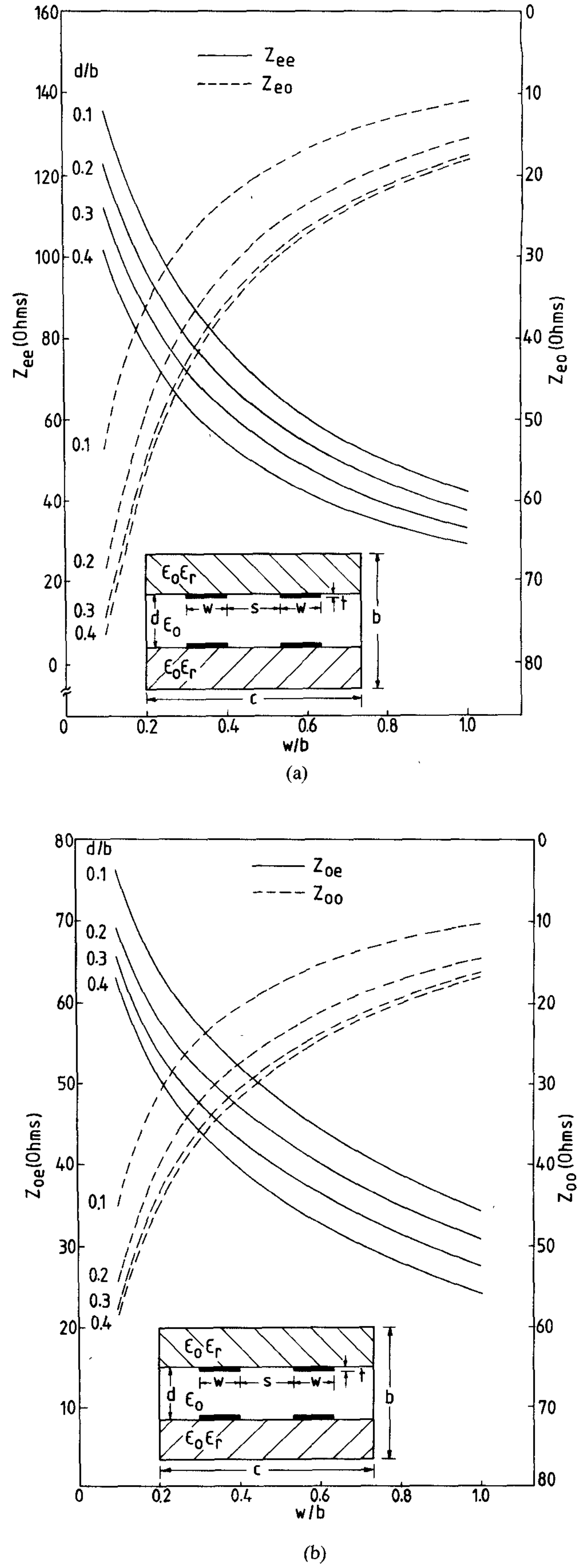


Fig. 5. The computed characteristic impedances of the broadside, edge-coupled, microstrip, for a fixed edge spacing $s, t/b = 0, c/b = 10, s/b = 0.2, \epsilon_r = 10$. (a) Even–even and even–odd mode impedances. (b) Odd–even and odd–odd mode impedances.

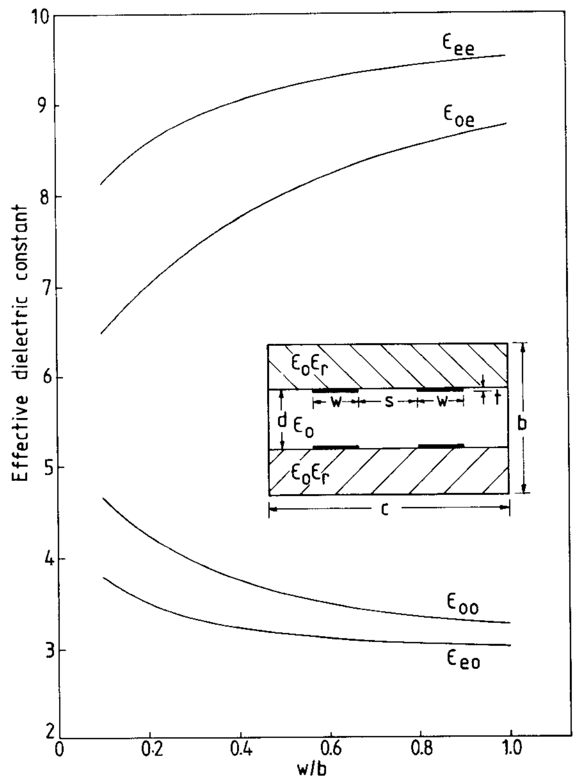


Fig. 6. The computed effective dielectric constants of the broadside, edge-coupled, microstrip, $s/b = d/b = 0.2, t/b = 0, c/b = 10, \epsilon_r = 10$.

with an increase in c/b is negligible. For all the computations, the shielding side walls are assumed to be sufficiently away ($c/b = 10$) so that they have negligible effect on the propagation characteristics.

Fig. 2(a) and (b) shows the computed characteristic impedances (Z_{ee}, Z_{eo}) and (Z_{oe}, Z_{oo}), respectively, of the broadside, edge-coupled, stripline with air dielectric, for a fixed value of d/b. Increasing w/b, for a fixed value of s/b, decreases all the four impedances. For a fixed value of w/b, Z_{ee} and Z_{eo} decrease, whereas Z_{oe} and Z_{oo} increase, with an increase in s/b. The variations in Z_{eo} and Z_{oo}, with increasing s/b, are much less as compared with those in Z_{ee} and Z_{oe}.

Fig. 3(a) and (b) illustrates the computed characteristic impedances (Z_{ee}, Z_{eo}) and (Z_{oe}, Z_{oo}), respectively, of the same structure for a fixed value of s/b. Increasing w/b, for a fixed value of d/b, decreases all the four impedances. For a fixed value of w/b, Z_{ee} and Z_{oe} decrease, while Z_{eo} and Z_{oo} increase, with an increase in d/b. It is observed that impedance values from 10 to 400 Ω are possible for the structural parameters in the range $0.1 \leqslant w/b \leqslant 1.0$, $0.2 \leqslant s/b \leqslant 1.0$, and $0.1 \leqslant d/b \leqslant 0.4$.

The computed characteristic impedances (Z_{ee}, Z_{eo}) and (Z_{oe}, Z_{oo}) of the broadside, edge-coupled, microstrip, for a fixed value of d/b, are plotted in Fig. 4(a) and (b), respectively. Fig. 5(a) and (b) shows the computed (Z_{ee}, Z_{eo}) and (Z_{oe}, Z_{oo}) of the same structure, for a fixed value of s/b, respectively. It is seen that the variations of Z_{ee}, Z_{eo}, Z_{oe} and Z_{oo} are similar to those obtained in the

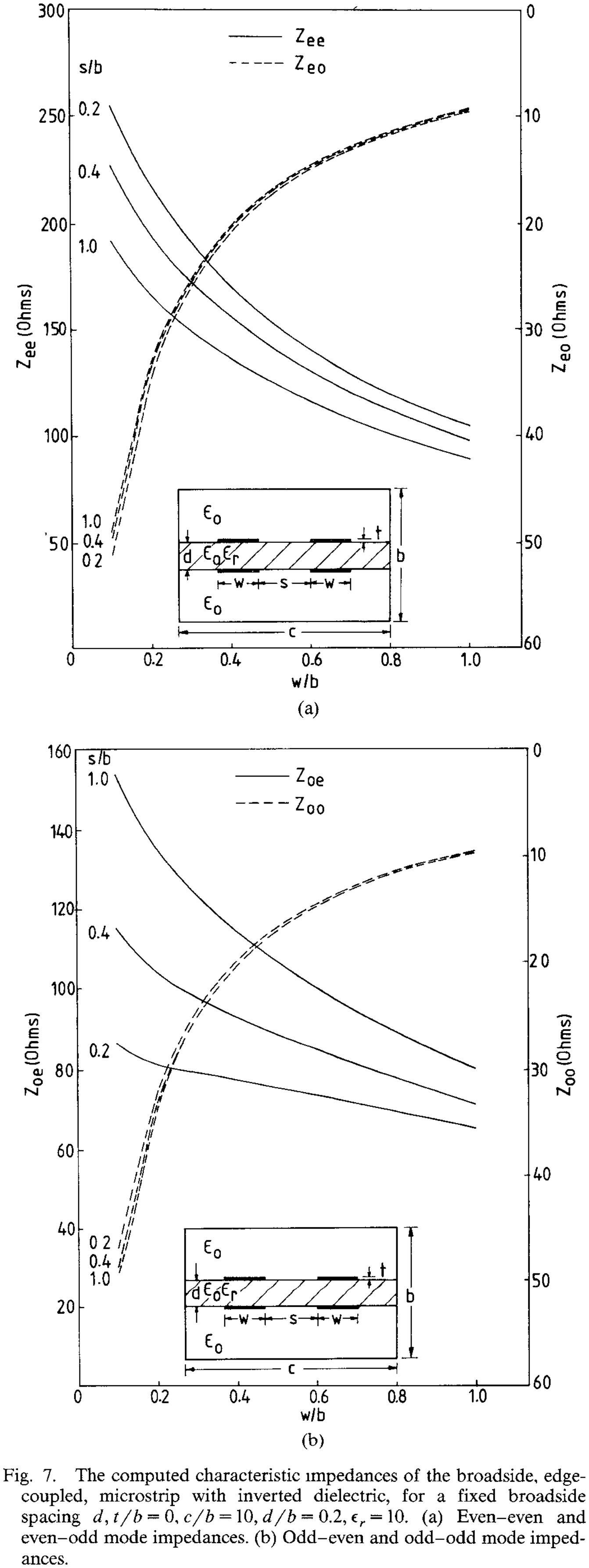


Fig. 7. The computed characteristic impedances of the broadside, edge-coupled, microstrip with inverted dielectric, for a fixed broadside spacing $d, t/b = 0, c/b = 10, d/b = 0.2, \epsilon_r = 10$. (a) Even–even and even–odd mode impedances. (b) Odd–even and odd–odd mode impedances.

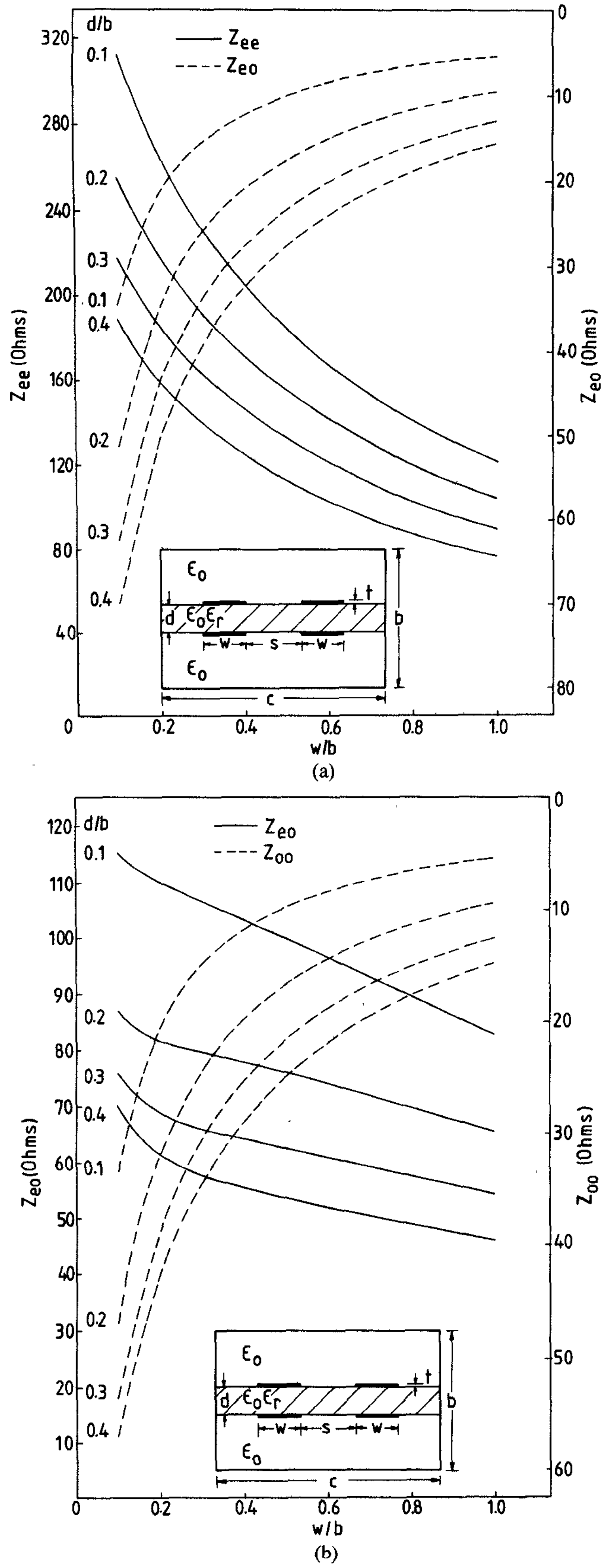


Fig. 8. The computed characteristic impedances of the broadside, edge-coupled, microstrip with inverted dielectric, for a fixed edge spacing s, $t/b = 0$, $c/b = 10$, $s/b = 0.2$, $\epsilon_r = 10$. (a) Even–even and even–odd mode impedances. (b) Odd–even and odd–odd mode impedances.

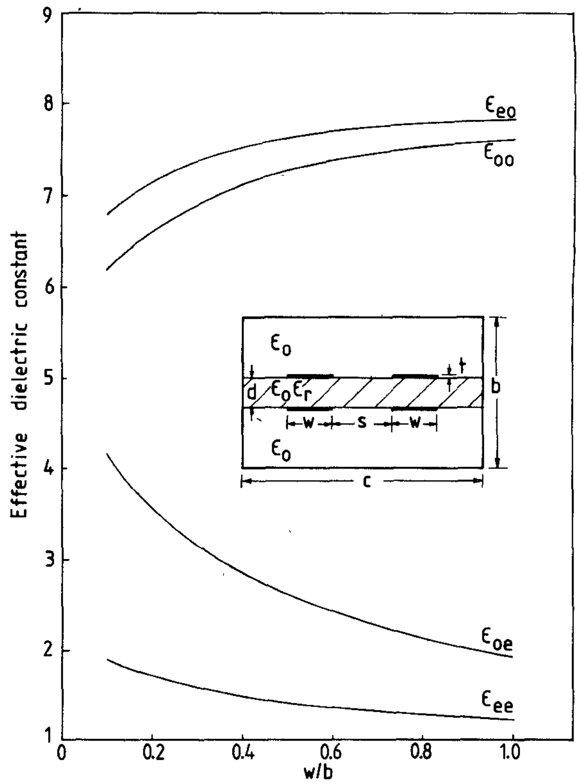


Fig. 9. The computed effective dielectric constants of the broadside, edge-coupled, microstrip with inverted dielectric, $s/b = d/b = 0.2$, $t/b = 0$, $c/b = 10$, $\epsilon_r = 10$.

broadside, edge-coupled, homogeneous stripline. For a dielectric of relative permittivity $\epsilon_r = 10$, the impedance values from 10 to 140 Ω are obtained for the structural parameters in the range $0.1 \leqslant w/b \leqslant 1.0$, $0.2 \leqslant s/b \leqslant 1.0$, and $0.1 \leqslant d/b \leqslant 0.4$. For fixed values of s/b and d/b, the computed effective dielectric constants of the broadside, edge-coupled, microstrip are plotted in Fig. 6. With an increase in w/b, the even–even and the odd–even mode effective dielectric constants ($\epsilon_{ee}, \epsilon_{oe}$) increase, while the even–odd and odd–odd mode effective dielectric constants ($\epsilon_{eo}, \epsilon_{oo}$) decrease. It is seen that the values of ϵ_{ee} and ϵ_{oe} are considerably greater than those of ϵ_{eo} and ϵ_{oo}.

The computed (Z_{ee}, Z_{eo}) and (Z_{oe}, Z_{oo}) of the broadside, edge-coupled, microstrip with inverted dielectric, for a fixed value of d/b, are plotted in Fig. 7(a) and (b), respectively. For a fixed value of s/b, the computed impedances (Z_{ee}, Z_{eo}) and (Z_{oe}, Z_{oo}) are illustrated in Fig. 8(a) and (b), respectively. It is observed that the variations of Z_{ee}, Z_{eo}, Z_{oe}, and Z_{oo} are similar to those obtained in the broadside, edge-coupled, microstrip. However, the variations in Z_{eo} and Z_{oo}, with increasing s/b, are negligible. For the structural parameters $0.1 \leqslant w/b \leqslant 1.0$, $0.2 \leqslant s/b \leqslant 1.0$, $0.1 \leqslant d/b \leqslant 0.4$, and $\epsilon_r = 10$, the impedance values obtained are in the range 10 to 320 Ω. The computed effective dielectric constants of the broadside edge-coupled microstrip with inverted dielectric are illustrated in Fig. 9. It is seen that, with an increase in w/b, ϵ_{eo} and ϵ_{oo} increase, while ϵ_{oe} and ϵ_{ee} decrease. The values of ϵ_{eo} and ϵ_{oo} are considerably greater than those of ϵ_{oe} and ϵ_{ee}.

IV. Conclusion

A simple technique to analyze the broadside, edge-coupled, strip transmission lines has been described. Extensive design data on the characteristic impedances and the effective dielectric constants for the broadside, edge-coupled, striplines in a homogeneous, as well as an inhomogeneous, medium are presented. It is observed that the values of the even–even and odd–even mode effective dielectric constants in the case of the broadside, edge-coupled, striplines in an inhomogeneous medium are considerably different from those of the even–odd and odd–odd mode effective dielectric constants. The effect of the shielding side walls on the impedance characteristics has been studied. The effect is more pronounced for the even–even and odd–even mode impedances than for the even–odd and odd–odd mode impedances. The accuracy of the method presented can further be enhanced by considering a more complex charge distribution on the strip conductor. This will, however, increase the computational time.

The data presented in this paper should be useful in designing directional couplers and filters for MIC application. A study of the electric response for various terminal conditions, physical dimensions, and dielectric constants should prove helpful in designing some new devices.

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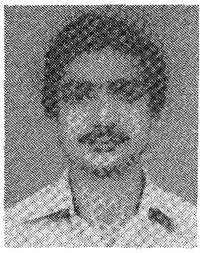

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